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Chapter 1

RATIONAL NUMBERS

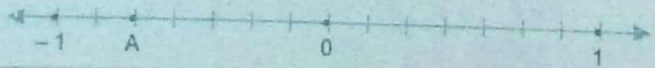
OBJECTIVE

In this chapter the students will be able to explore :

- Rational numbers.
- Properties of rational numbers : closure property, commutative property, associative property, distributive property.
- Additive identity and multiplicative identity.
- Additive inverse and multiplicative inverse.
- Rational numbers on number line.
- Rational numbers between two given rational numbers.
- Word problems.

WARM UP!!

1. Write $\frac{156}{-78}$ in standard form = ...
2. $-\left|\frac{-15}{27}\right|$ = ...
3. Which of the two rational numbers is greater? $\frac{3}{-5}$ or $\frac{-5}{6}$
4. $\left(-\frac{2}{7}\right)+\left(\frac{-5}{9}\right)$ = ...
5. $-\frac{1}{21}-\left(\frac{-3}{7}\right)$ = ...
6. $2\frac{2}{7}\times1\frac{5}{8}$ = ...
7. $\frac{10}{13}\div1\frac{2}{3}$ = ...
8. Express $\frac{1}{50}$ as a decimal.
9. Reciprocal of $-2\frac{1}{8}$ is ....
10. Rational number represented by a point A on the number line is



INTRODUCTION

We have already dealt with natural numbers, whole numbers, integers, fractions and rational numbers. We have also learnt about various operations on rational numbers. In this chapter, we shall discuss the properties of rational numbers and strengthen our knowledge and technique of dealing with rational numbers.

RATIONAL NUMBERS

The numbers which can be put in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

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Examples : $\frac{0}{1}, \frac{3}{3}, \frac{4}{5}, \frac{-6}{11}, \frac{-7}{-9}$

Numerators and denominators of each of the given numbers are integers. Thus, all such numbers are called **Rational Numbers**.

REMEMBER!

$\frac{3}{0}, \frac{1}{0}, \frac{-9}{0}$, etc. are not rational numbers because division by zero is not possible.

1. Zero is a rational number *e.g.*, $0 = \frac{0}{1}$
2. Every integer is a rational number.

NOTE :

Natural numbers, whole numbers, integers and decimal numbers are not in the form of $\frac{p}{q}$, but can be written in the form $\frac{p}{q}$ and as such are rational numbers.

e.g.,

$8 = \frac{8}{1}$; $\quad 0 = \frac{0}{1}$;

$-6 = \frac{-6}{1}$ $\quad 4.2 = \frac{42}{10}$

3. Every fraction is a rational number *e.g.*, $\frac{3}{2}, \frac{1}{6}$

All integers, together with fractions (including decimals) are called rational numbers.

(*a*) **POSITIVE RATIONAL NUMBERS**

$\frac{7}{9}, \frac{5}{11}, \frac{-2}{-3}, \frac{-38}{-117}$

Numerator and Denominator are either both + *ve* or both – *ve*

(*b*) **NEGATIVE RATIONAL NUMBERS**

$\frac{-7}{8}, \frac{5}{-31}, \frac{-99}{100}$

Either Numerator or Denominator is – *ve*

Let us Recall

1. **Equivalent Rational Numbers.** If $\frac{p}{q}$ is a rational number and m is a non-zero integer then $\frac{p}{q} = \frac{p \times m}{q \times m}$

e.g.,
$$\frac{-5}{6} = \frac{-5\times 2}{6\times 2} = \frac{-5\times 3}{6\times 3} = \frac{-5\times 4}{6\times 4} = ...$$

Hence,
$$\frac{-5}{6} = \frac{-10}{12} = \frac{-15}{18} = \frac{-20}{24} = ...$$

Such rational numbers are called **Equivalent Rational Numbers.**

2. **Standard Form of a Rational Number.** A rational number $\frac{p}{q}$ is said to be in **standard form** if q is positive and p and q have no common divisor other than 1.

e.g., $\frac{56}{-77} = \frac{56 \div (-7)}{-77 \div (-7)} = \frac{-8}{11}$ (Standard form)

(Since H.C.F. of 56 and 77 is 7)

'D' should be Positive

H.C.F. of 'N' and 'D' should be 1

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OPERATIONS ON RATIONAL NUMBERS

If $\frac{p}{q}$ and $\frac{r}{s}$ are two rational numbers then

(i) $\frac{p}{q}+\frac{r}{s}=\frac{ps+qr}{qs}$

e.g., $-\frac{8}{9}+\frac{2}{45}=\frac{-8\times5+2\times1}{45}=\frac{-40+2}{45}=\frac{-38}{45}$

(ii) $\frac{p}{q}-\frac{r}{s}=\frac{ps-qr}{qs}$

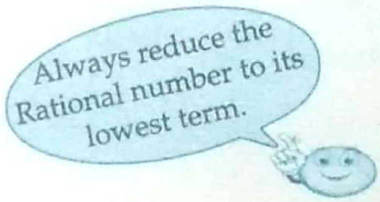
e.g., $\frac{4}{5}-\frac{6}{7}=\frac{4\times7-6\times5}{35}=\frac{28-30}{35}=\frac{-2}{35}$

(iii) $\frac{p}{q}\times\frac{r}{s}=\frac{p\times r}{q\times s}$

e.g., $\frac{5}{6}\times\frac{4}{7}=\frac{5\times \cancel{4}^{2}}{\cancel{6}^{3}\times7}=\frac{10}{21}$

(iv) $\frac{p}{q}\div\frac{r}{s}=\frac{p}{q}\times\frac{s}{r}=\frac{p\times s}{q\times r}$

e.g., $\frac{4}{27}\div\frac{8}{9}=\frac{4}{27}\times\frac{9}{8}=\frac{\cancel{4}^{1}\times\cancel{9}}{\cancel{27}_{3}\times\cancel{8}_{2}}=\frac{1}{6}$



PROPERTIES OF RATIONAL NUMBERS

1. Closure Property

(a) $\frac{5}{6}+\frac{3}{5}=\frac{25+18}{30}=\frac{43}{30}$ is a rational number

(b) $-\frac{1}{2}+\frac{2}{7}=\frac{-7+4}{14}=\frac{-3}{14}$ is a rational number

(c) $\frac{7}{9}-\frac{3}{11}=\frac{77-27}{99}=\frac{50}{99}$ is a rational number

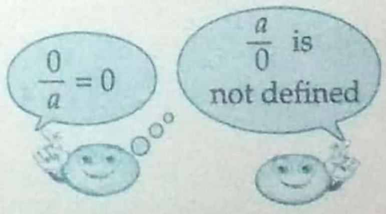
(d) $\frac{1}{6}-\frac{2}{9}=\frac{3-4}{18}=-\frac{1}{18}$ is a rational number

(e) $\frac{2}{5}\times\frac{3}{11}=\frac{2\times3}{5\times11}=\frac{6}{55}$ is a rational number

(f) $\frac{-3}{8}\times\frac{4}{9}=\frac{-\cancel{3}^{-1}\times\cancel{4}}{\cancel{8}_{2}\times\cancel{9}_{3}}=\frac{-1}{6}$ is a rational number

(g) $\frac{-5}{8}\div\frac{85}{6}=\frac{-5}{8}\times\frac{6}{85}=\frac{-\cancel{5}^{1}\times\cancel{6}^{3}}{\cancel{8}_{4}\times\cancel{85}_{17}}=\frac{-3}{68}$ is a rational number

(h) $\frac{-11}{42}\div\frac{33}{-7}=\frac{-11}{42}\times\frac{-7}{33}=\frac{-\cancel{11}^{-1}\times-\cancel{7}^{-1}}{\cancel{42}_{6}\times\cancel{33}_{3}}=\frac{1}{18}$ is a rational number



NOTE

Division by ZERO is not POSSIBLE.

Thus, sum, difference, product and quotient of rational numbers is a rational number.

We can say that

Rational numbers are closed under addition, subtraction, multiplication and division (excluding division by 0)

2. Commutative Property

(a) **Addition**

$$\frac{1}{2}+\frac{4}{5}=\frac{5+8}{10}=\frac{13}{10}$$

$$\frac{4}{5}+\frac{1}{2}=\frac{8+5}{10}=\frac{13}{10}$$

So, $\frac{1}{2}+\frac{4}{5}=\frac{4}{5}+\frac{1}{2}$

(b) **Subtraction**

Is $\frac{5}{7}-\frac{3}{7}=\frac{3}{7}-\frac{5}{7}$?

You will observe that $\frac{5}{7}-\frac{3}{7}\neq\frac{3}{7}-\frac{5}{7}$

(c) **Multiplication**

$$\frac{3}{8}\times\frac{5}{7}=\frac{15}{56} \text{ and } \frac{5}{7}\times\frac{3}{8}=\frac{15}{56}$$

So, $\frac{3}{8}\times\frac{5}{7}=\frac{5}{7}\times\frac{3}{8}$

(d) **Division**

Is $\frac{1}{2}\div\frac{-3}{4}=\frac{-3}{4}\div\frac{1}{2}$?

You will observe that $\frac{1}{2}\div\frac{-3}{4}\neq\frac{-3}{4}\div\frac{1}{2}$

We can say that

Commutative property holds for addition and multiplication of rational numbers only. This property does not hold for subtraction and division of rational numbers.

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers then,

$$\frac{p}{q}+\frac{r}{s}=\frac{r}{s}+\frac{p}{q} \quad \text{and} \quad \frac{p}{q}\times\frac{r}{s}=\frac{r}{s}\times\frac{p}{q}$$

. Associative Property

(a) **Addition**

$$\frac{1}{2}+\left(\frac{-4}{5}+\frac{2}{3}\right)=\frac{1}{2}+\left(\frac{-12+10}{15}\right)=\frac{1}{2}+\frac{-2}{15}=\frac{15+(-4)}{30}=\frac{11}{30}$$

$$\left(\frac{1}{2}+\frac{-4}{5}\right)+\frac{2}{3}=\left(\frac{5+(-8)}{10}\right)+\frac{2}{3}=\frac{-3}{10}+\frac{2}{3}=\frac{-9+20}{30}=\frac{11}{30}$$

Hence, $\frac{1}{2}+\left(\frac{-4}{5}+\frac{2}{3}\right)=\left(\frac{1}{2}+\frac{-4}{5}\right)+\frac{2}{3}$

(*b*) **Subtraction**

Is $\frac{-1}{3}-\left(\frac{6}{5}-\frac{2}{3}\right)=\left(-\frac{1}{3}-\frac{6}{5}\right)-\frac{2}{3}$?

You will observe that

$$\frac{-1}{3}-\left(\frac{6}{5}-\frac{2}{3}\right)\neq\left(\frac{-1}{3}-\frac{6}{5}\right)-\frac{2}{3}$$

(*c*) **Multiplication**

$$\left(\frac{5}{7}\times\frac{-3}{4}\right)\times\frac{1}{2}=\frac{-15}{28}\times\frac{1}{2}=\frac{-15}{56}$$

$$\frac{5}{7}\times\left(\frac{-3}{4}\times\frac{1}{2}\right)=\frac{5}{7}\times\frac{-3}{8}=\frac{-15}{56}$$

Hence, $\left(\frac{5}{7}\times\frac{-3}{4}\right)\times\frac{1}{2}=\frac{5}{7}\times\left(\frac{-3}{4}\times\frac{1}{2}\right)$

(*d*) **Division**

Is $\frac{-4}{5}\div\left(\frac{6}{5}\div\frac{3}{2}\right)=\left(\frac{-4}{5}\div\frac{6}{5}\right)\div\frac{3}{2}$?

You will observe that $\frac{-4}{5}\div\left(\frac{6}{5}\div\frac{3}{2}\right)\neq\left(\frac{-4}{5}\div\frac{6}{5}\right)\div\frac{3}{2}$

We can say that

Associative Property holds for addition and multiplication of rational numbers. This property does not hold for subtraction and division of rational numbers.

Hence if, $\frac{p}{q},\frac{r}{s}$ and $\frac{t}{u}$ are rational numbers, then

$$\frac{p}{q}+\left(\frac{r}{s}+\frac{t}{u}\right)=\left(\frac{p}{q}+\frac{r}{s}\right)+\frac{t}{u}$$

and

$$\frac{p}{q}\times\left(\frac{r}{s}\times\frac{t}{u}\right)=\left(\frac{p}{q}\times\frac{r}{s}\right)\times\frac{t}{u}$$

Did you know??

This property of Rational Numbers is also called **Re-arrangement Property for Addition** of Rational Numbers.

4. Distributive Property of Multiplication over Addition for Rational Numbers

$$\frac{4}{5}\times\left(\frac{2}{3}+\frac{7}{15}\right)=\frac{4}{5}\times\left(\frac{10+7}{15}\right)=\frac{4}{5}\times\frac{17}{15}=\frac{68}{75}$$

$$\frac{4}{5}\times\frac{2}{3}+\frac{4}{5}\times\frac{7}{15}=\frac{8}{15}+\frac{28}{75}=\frac{40+28}{75}=\frac{68}{75}$$

Thus, $\frac{4}{5}\times\left(\frac{2}{3}+\frac{7}{15}\right)=\frac{4}{5}\times\frac{2}{3}+\frac{4}{5}\times\frac{7}{15}$

Hence, if $\frac{p}{q},\frac{r}{s}$ and $\frac{t}{u}$ are any three rational numbers, then

$$\frac{p}{q}\left(\frac{r}{s}+\frac{t}{u}\right)=\frac{p}{q}\times\frac{r}{s}+\frac{p}{q}\times\frac{t}{u}$$

5. Property of Zero

$$\frac{4}{5}+0=\frac{4}{5}$$

$$\frac{-2}{7}+0=\frac{-2}{7}$$

What happens when you add 0 to any rational number? You will observe that when you add 0 to any rational number, the sum is the rational number itself.

0 is called the additive identity.

Hence, if $\frac{p}{q}$ is any rational number

then, $$\frac{p}{q} + 0 = 0 + \frac{p}{q} = \frac{p}{q}$$

6. Property of One

$$\frac{5}{7} \times 1 = \frac{5}{7}$$

$$\frac{-2}{3} \times 1 = \frac{-2}{3}$$

What happens when you multiply 1 by any rational number? You will observe that when you multiply any rational number by 1, you get the product as the rational number itself.

1 is called the multiplicative identity.

$$\frac{p}{q} \times 1 = 1 \times \frac{p}{q} = \frac{p}{q}$$

7. Negative of a Number (Existence of Additive Inverse)

$$\frac{2}{3} + \left(\frac{-2}{3}\right) = 0$$

$$\frac{-5}{6} + \frac{5}{6} = 0$$

If $\frac{p}{q}$ is any rational number then there exists another rational number $-\frac{p}{q}$ such that

$$\frac{p}{q} + \left(\frac{-p}{q}\right) = \left(\frac{-p}{q}\right) + \frac{p}{q} = 0$$

NOTE : $\frac{p}{q}$ and $-\frac{p}{q}$ are called the additive inverse (or negative) of each other.

e.g., Additive Inverse of $\frac{-6}{5}$ is $\frac{6}{5}$

Additive Inverse of $\frac{5}{11}$ is $\frac{-5}{11}$

Did you know?? Additive inverse of zero is zero.

8. Reciprocal of a Number (Multiplicative Inverse)

NOTE

Multiplicative Inverse of '0' does not exist

$$\frac{7}{3} \times \frac{3}{7} = 1$$

$$\frac{-5}{2} \times \frac{-2}{5} = 1$$

If $\frac{p}{q}$ is any rational number then there exists another rational number $\frac{q}{p}$ such that

$$\frac{p}{q} \times \frac{q}{p} = \frac{q}{p} \times \frac{p}{q} = 1$$

NOTE! $\frac{p}{q}$ and $\frac{q}{p}$ are called the multiplicative inverse (or reciprocal) of each other.

e.g., Multiplicative inverse of $\frac{5}{3}$ is $\frac{3}{5}$

Reciprocal of $\frac{-6}{11}$ is $\frac{-11}{6}$

Did you know??

1 and – 1 are the only rational numbers which are their own reciprocals.

REMEMBER!

(i) For a Rational number $\frac{p}{q}$, the additive identity is '0' and multiplicative identity is '1'.

(ii) For a Rational number $\frac{p}{q}$, the additive inverse is $\frac{-p}{q}$ and multiplicative inverse is $\frac{q}{p}$.

Now let us consider some examples :

Example 1 : Simplify using properties

$$\frac{4}{5}+\left(\frac{8}{-7}\right)+\left(\frac{-13}{5}\right)+\frac{8}{7}$$

Solution :

$$\frac{4}{5}+\left(\frac{-8}{7}\right)+\left(\frac{-13}{5}\right)+\frac{8}{7} = \frac{4}{5}+\left(\frac{-13}{5}\right)+\left(\frac{-8}{7}\right)+\frac{8}{7} \quad \text{(Commutative property of addition)}$$

$$= \left[\frac{4}{5}+\left(\frac{-13}{5}\right)\right]+\left[\left(\frac{-8}{7}\right)+\frac{8}{7}\right]$$

$$= \frac{4+(-13)}{5}+0$$

$$= \frac{-9}{5}+0$$

$$= \frac{-9}{5} \quad \text{(Additive property of zero)}$$

NOTE!!

If the 'Denominator' of a Rational number is negative, re-write it so that 'D' becomes positive.

e.g., $\frac{8}{-7} = \frac{-8}{7}$

Example 2 : Simplify using properties

$$\frac{-5}{3}\times\frac{-9}{10}\times\frac{4}{21}\times\frac{-1}{6}$$

Solution :

$$\frac{-5}{3}\times\frac{-9}{10}\times\frac{4}{21}\times\frac{-1}{6} = \left[\left(\frac{-5}{3}\right)\times\left(\frac{-9}{10}\right)\right]\times\left[\frac{4}{21}\times\left(\frac{-1}{6}\right)\right]$$

$$= \frac{-5\times(-9)}{3\times10}\times\frac{4\times(-1)}{21\times6} = \frac{3}{2}\times\frac{-2}{63} = \frac{-1}{21}$$

Example 3 : What should be subtracted from $\frac{-5}{9}$ to get $\frac{1}{6}$?

Solution : Let the required number to be subtracted be x

$$\frac{-5}{9} - x = \frac{1}{6}$$

$\frac{-5}{9} - \square = \frac{1}{6}$

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$$\frac{-5}{9}=\frac{1}{6}+x$$

$$x=\frac{-5}{9}-\frac{1}{6}$$

$$x=\frac{-30-9}{54}=\frac{-39}{54}=\frac{-13}{18}$$

Hence, $\frac{-13}{18}$ should be subtracted from $\frac{-5}{9}$ to get $\frac{1}{6}$

Example 4 : (*a*) Write the multiplicative inverse of $\frac{-6}{5}\times\frac{2}{-3}$

(*b*) Write the additive inverse of $\frac{-5}{6}+\frac{2}{3}$

Solution : (*a*) $\frac{-6}{5}\times\frac{2}{-3}=\frac{4}{5}$

Multiplicative inverse of $\frac{4}{5}$ is $\frac{5}{4}$

(*b*) $\frac{-5}{6}+\frac{2}{3}=\frac{-5+4}{6}=\frac{-1}{6}$

Additive inverse of $\frac{-1}{6}$ is $\frac{1}{6}$

Example 5 : Simplify $\frac{2}{13}\times\frac{-5}{7}+\frac{2}{13}\times\frac{1}{3}$

Solution : $\frac{2}{13}\times\frac{-5}{7}+\frac{2}{13}\times\frac{1}{3}=\frac{2}{13}\times\left(\frac{-5}{7}+\frac{1}{3}\right)$

$$=\frac{2}{13}\times\left(\frac{-15+7}{21}\right)=\frac{2}{13}\times\frac{-8}{21}=\frac{-16}{273}$$

Example 6 : Using distributive property, evaluate $\frac{-5}{3}\times\frac{5}{7}-\frac{4}{7}\times\frac{5}{3}$

Solution : $\frac{-5}{3}\times\frac{5}{7}-\frac{4}{7}\times\frac{5}{3}=\frac{-5}{3}\times\frac{5}{7}-\frac{5}{3}\times\frac{4}{7}$

$$=\frac{-5}{3}\times\left(\frac{5}{7}+\frac{4}{7}\right)$$

$$=\frac{-5}{3}\times\frac{9}{7}$$

$$=\frac{-5\times \cancel{9}^{3}}{\cancel{3}\times 7}=\frac{-15}{7}$$

Example 7 : Divide $\frac{-25}{48}$ by $\frac{15}{6}$

Solution : $\frac{-25}{48}\div\frac{15}{6}=\frac{\cancel{-25}^{-5}}{\cancel{48}_{24}}\times\frac{\cancel{6}^{2}}{\cancel{15}_{3}}=\frac{-5}{24}$

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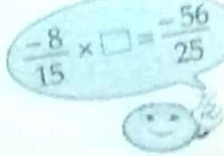
Example 8 : The product of two rational numbers is $\frac{-56}{25}$. If one number is $\frac{-8}{15}$, find the other.

Solution : Product of two rational numbers = $\frac{-56}{25}$



One number = $\frac{-8}{15}$

Other number = x

$$\frac{-8}{15} \times x = \frac{-56}{25}$$

$$x = \frac{-56}{25} \div \frac{-8}{15} = \frac{-56}{25} \times \frac{15}{-8}$$

$$= \frac{-56 \times 15}{25 \times -8} = \frac{-21}{-5} = \frac{21}{5}$$

Hence, the other number is $\frac{21}{5}$

Example 9 : Divide the sum of $\frac{-2}{3}$ and $\frac{5}{4}$ by their difference.

Solution : Sum : $\frac{-2}{3} + \frac{5}{4} = \frac{-8+15}{12} = \frac{7}{12}$

Difference : $\frac{-2}{3} - \frac{5}{4} = \frac{-8-15}{12} = \frac{-23}{12}$

Hence, division = $\frac{7}{12} \div \left(\frac{-23}{12}\right) = \frac{7}{12} \times \left(\frac{12}{-23}\right) = \frac{-7}{23}$

EXERCISE 1.1

1. Simplify

(*a*) $\frac{4}{7} + \frac{2}{21} + \frac{1}{3}$ (*b*) $\frac{-7}{9} - \left(\frac{-5}{12}\right) + \frac{1}{3}$ (*c*) $\frac{7}{9} + \left(\frac{-2}{3}\right) + \left(\frac{-11}{18}\right) + \frac{1}{6}$

(*d*) $\frac{-7}{22} \times \frac{-14}{35} \times \frac{-12}{5}$ (*e*) $\frac{-4}{5} \times \frac{5}{7} \times \left(\frac{-8}{9}\right) + \frac{8}{9} \times \frac{4}{7}$ (*f*) $\frac{-3}{2} \times \frac{-6}{5} + \frac{2}{3} + \frac{5}{6} + \frac{7}{6}$

2. Write the multiplicative inverse of the following :

(*a*) $\frac{-16}{23}$ (*b*) -13 (*c*) $\frac{3}{11} \times \frac{-6}{5}$

3. Write the additive inverse of the following :

(*a*) $\frac{-6}{13}$ (*b*) -8 (*c*) $\frac{-16}{-31}$ (*d*) $\frac{-5}{13} \times \frac{-26}{15}$

4. What number should be added to $\frac{-3}{8}$ to get $\frac{7}{9}$?

5. Subtract the sum of $\frac{-5}{8}$ and $\frac{7}{10}$ from the sum of $\frac{3}{-5}$ and $\frac{8}{15}$

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6. A piece of wire $\frac{15}{4}$ m long is broken into two pieces. One piece is $2\frac{1}{2}$ m long. Find the length of the other piece.

7. By what number should we multiply $\frac{-12}{13}$ to get $\frac{4}{39}$?

8. Divide the sum of $\frac{11}{7}$ and $\frac{-7}{5}$ by their product.

9. Divide the sum of $\frac{-9}{4}$ and $\frac{-8}{3}$ by the difference of $\frac{13}{8}$ and $\frac{-7}{16}$

10. The cost of $5\frac{2}{7}$ metres of cloth is ₹ $28\frac{1}{3}$. What is the cost of 1 metre of cloth?

11. Find the area of a square piece of land whose each side measures $6\frac{1}{4}$ m.

12. The area of a rectangle is $45\frac{1}{2}$ m^2. If its length is $3\frac{1}{4}$ m, what is its breadth?

13. Using distributive law, evaluate :

(*a*) $\frac{3}{7}\times\frac{7}{3}+\frac{3}{7}\times\frac{5}{3}$ (*b*) $\frac{4}{7}\times\frac{10}{9}-\frac{4}{7}\times\frac{1}{9}$

(*c*) $\frac{-4}{5}\times\frac{5}{8}+\frac{4}{5}\times\left(\frac{-7}{8}\right)$ (*d*) $\frac{5}{11}\times\left(\frac{-6}{11}\right)+(-4)\times\frac{5}{11}$

14. Name the property illustrated by the following :

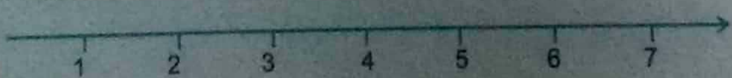
(*a*) $\frac{-5}{7}\times\frac{3}{10}=\frac{3}{10}\times\frac{-5}{7}$ (*b*) $\frac{-5}{9}\times\left(\frac{-6}{13}+\frac{5}{26}\right)=\left(\frac{-5}{9}\times\frac{-6}{13}\right)+\left(\frac{-5}{9}\times\frac{5}{26}\right)$

(*c*) $\frac{-6}{13}\times\frac{-13}{6}=1$ (*d*) $\frac{-5}{3}+\frac{6}{5}=\frac{6}{5}+\frac{-5}{3}$ (*e*) $\frac{-2}{3}+\left(\frac{5}{6}-\frac{7}{9}\right)=\left(\frac{-2}{3}+\frac{5}{6}\right)-\frac{7}{9}$

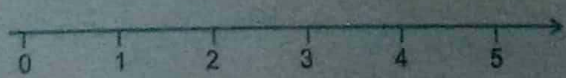
REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

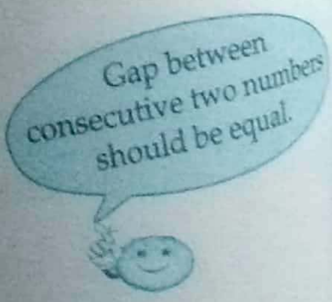
Let us recall the representation of natural numbers, whole numbers, integers and rational numbers on number line.

(*a*) Natural Numbers



(*b*) Whole Numbers





(*c*) Integers

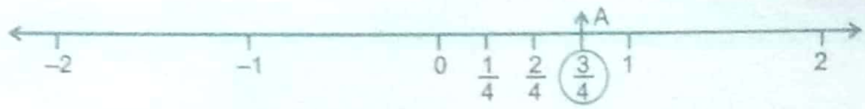
(*d*) Rational Numbers

Here, the gap between 0 and 1 or 0 and –1 is divided into 2 subparts and each subpart represents a rational number $\frac{1}{2}$ and $\frac{-1}{2}$ respectively.

Example 10 : Represent $\frac{3}{4}$ on number line.

Solution : If the gap between 0 and 1 is divided into 4 subparts of equal size then each part is equal to the length of a rational number $\frac{1}{4}$. Two such parts combined, represent the rational number $\frac{2}{4}$ and three parts represent $\frac{3}{4}$ and so on.

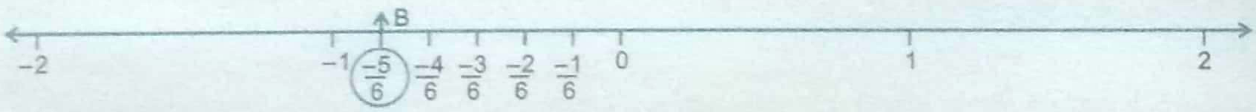
The rational number $\frac{3}{4}$ is shown on the number line drawn below :



Example 11 : Represent $\frac{-5}{6}$ on the number line.

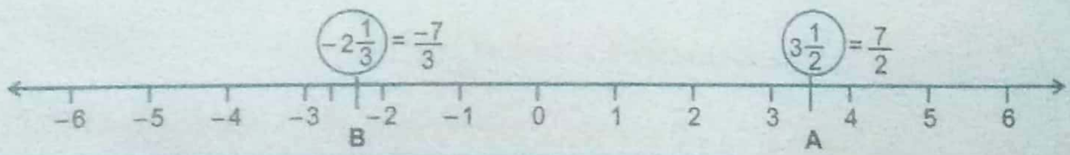
Solution : If the gap between 0 and –1 is divided into 6 equal parts, then each part is of length equal to the rational number $-\frac{1}{6}$, two together represent $-\frac{2}{6}$, three $-\frac{3}{6}$ and so on.

Rational number $\frac{-5}{6}$ is shown on the following number line :



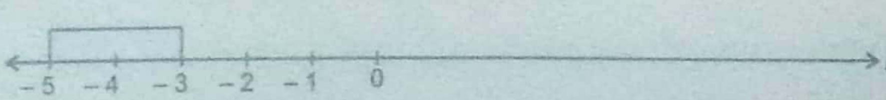
Example 12 : Represent $\frac{7}{2}$ and $\frac{-7}{3}$ on the number line.

Solution : Rational numbers $\frac{7}{2}$ and $\frac{-7}{3}$ are represented on the number line by points 'A' and 'B' as shown below :



RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

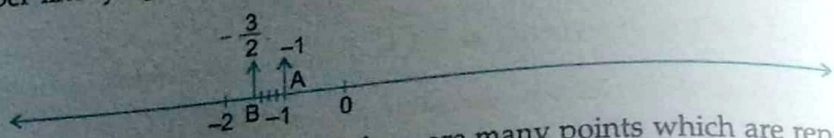
If we want to write the number of integers lying between –3 and –5, obviously there is only one number *i.e.*, – 4, as is clear from the following number line :



Similarly , we can say that 4 integers lie between –2 and 3

As $-2 < -1 < 0 < 1 < 2 < 3$, the numbers are –1, 0, 1, 2

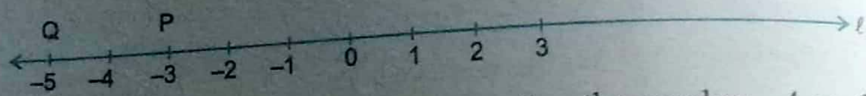
Now, if we want to find the number of rational numbers lying between $\frac{-3}{2}$ and –1, then we find difficult to get the correct number as the number line given below clearly shows that there are ma points on the number line lying between A and B.



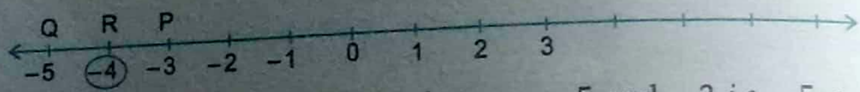
Out of these points (lying between A and B) there are many points which are representing the rati numbers.

Hence, we can say that **there are infinite (countless) number of numbers lying between any rational numbers.**

To justify this property of rational numbers, let us consider two rational numbers, say – 3 and – a number line and name the points corresponding to these numbers as P and Q.



On bisecting $\overline{PQ}$ we get the point R which is representing the number – 4 on the number line



From this number line, we observe that – 4 lies between – 5 and – 3 *i.e.*, –5 < –4 < –3.

or $-5 < \frac{(-5)+(-3)}{2} < -3$

Let us again bisect $\overline{PR}$ and name the point of bisection as S.

Clearly, point S is representing the rational number $-3\frac{1}{2}$ lying between 'R' and 'P'.

Number line shows that $-5 < -4 < -3\frac{1}{2} < -3$

or $-5 < \frac{(-5)+(-3)}{2} < \frac{(-4)+(-3)}{2} < -3$

Proceeding repeatedly in this manner, we may say that :

(*i*) Many rational numbers are lying between –3 and –5. In general we may state that there ar **rational numbers between two rational numbers $\frac{p}{q}$ and $\frac{r}{s}$**

(*ii*) Number $\frac{\frac{p}{q}+\frac{r}{s}}{2}$ always lies between $\frac{p}{q}$ and $\frac{r}{s}$

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REMEMBER!

$\dfrac{\frac{p}{q}+\frac{r}{s}}{2}$ is the Arithmetic mean of $\frac{p}{q}$ and $\frac{r}{s}$. Thus, we can say that between two rational numbers, there lie an infinite number of rational numbers. This is known as the density property of rational numbers.

Example 13 : Find three rational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$

Solution : First rational number $= \frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{2+3}{6}\right)$

$$= \frac{1}{2}\times\frac{5}{6} = \frac{5}{12}$$

Second number $= \frac{1}{2}\left(\frac{1}{3}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{4+5}{12}\right) = \frac{1}{2}\times\frac{9}{12} = \frac{1}{2}\times\frac{3}{4} = \frac{3}{8}$

Third number $= \frac{1}{2}\left(\frac{1}{2}+\frac{5}{12}\right) = \frac{1}{2}\left(\frac{6+5}{12}\right) = \frac{1}{2}\times\frac{11}{12} = \frac{11}{24}$ and so on.

Thus, three numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$ are $\frac{5}{12}, \frac{3}{8}$ and $\frac{11}{24}$

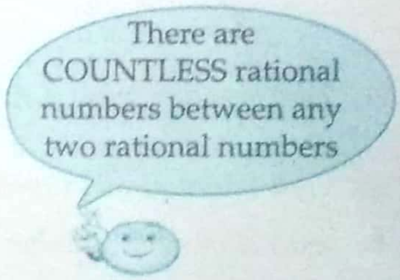
Alternative Method

Let us find some rational numbers between $\frac{-3}{8}$ and $\frac{3}{8}$. Clearly, the numbers are $\frac{-2}{8}, \frac{-1}{8}, \frac{0}{8}, \frac{1}{8}$ and $\frac{2}{8}$

But, this is not the limit. We can write more rational numbers between $\frac{-3}{8}$ and $\frac{3}{8}$.

We know that, $\frac{-3}{8} = \frac{-30}{80}$ and $\frac{3}{8} = \frac{30}{80}$

Hence, Rational numbers lying between $\frac{-30}{80}$ and $\frac{30}{80}$ are $\frac{-29}{80}, \frac{-29}{80}, \frac{-27}{80} \ldots \frac{0}{80}, \frac{1}{80}, \frac{2}{80}, \frac{3}{80} \ldots$ up to $\frac{29}{80}$. Similarly, by writing $\frac{-3}{8} = \frac{-300}{800}$ and $\frac{3}{8} = \frac{300}{800}$, we can write many other rational numbers between $\frac{-3}{8}$ and $\frac{3}{8}$



Hence, we conclude that

Example 14 : Find ten rational numbers between $\frac{4}{5}$ and $\frac{-3}{4}$

Solution : L.C.M. of 5 and 4 is 20

Hence, $\frac{4}{5} = \frac{16}{20}$ and $\frac{-3\times5}{4\times5} = \frac{-15}{20}$

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Now 10 Rational Numbers between $\frac{4}{5}$ and $\frac{-3}{4}$ or $\frac{16}{20}$ and $\frac{-15}{20}$ are :

$\frac{15}{20}, \frac{14}{20}, \frac{13}{20}, \frac{12}{20}, \frac{10}{20}, \frac{9}{20}, \frac{8}{20}, \frac{7}{20}, \frac{6}{20}$

NOTE : There are more rational numbers between $\frac{4}{5}$ and $\frac{-3}{4}$

Example 15 : Find six rational numbers between $\frac{-2}{3}$ and $\frac{-5}{6}$.

Solution : L.C.M of 3 and 6 is 6

Therefore, $\frac{-2}{3} = \frac{-4}{6}$ and $\frac{-5}{6} = \frac{-5}{6}$

Also, $\frac{-4}{6} = \frac{-40}{60}$ and $\frac{-5}{6} = \frac{-50}{60}$

Hence, 6 Rational Numbers between $\frac{-2}{3}$ and $\frac{-5}{6}$ or $\frac{-40}{60}$ and $\frac{-50}{60}$ are

$\frac{-41}{60}, \frac{-42}{60}, \frac{-43}{60}, \frac{-44}{60}, \frac{-45}{60}$ and $\frac{-46}{50}$

Example 16 : Find four rational numbers greater than 1

Solution : Rational numbers greater than 1 are

$\frac{5}{4}, \frac{6}{5}, \frac{8}{5}, \frac{10}{9}$

(As Numerator > Denominator in all the cases, thus these numbers are greater than 1)

EXERCISE 1.2

1. Represent the following rational numbers on the number line :

   (*a*) $\frac{-3}{5}$ (*b*) $\frac{4}{7}$ (*c*) $\frac{-8}{3}$ (*d*) $\frac{9}{4}$

2. Find three rational numbers between $\frac{-2}{7}$ and $\frac{2}{7}$
3. Find four rational numbers between $\frac{-1}{3}$ and $\frac{4}{9}$
4. Find a rational number between 3 and 4.
5. Find six rational numbers between $\frac{2}{9}$ and $\frac{5}{8}$
6. Find five rational numbers between $-\frac{1}{4}$ and $\frac{3}{8}$
7. Find ten rational numbers between $\frac{2}{3}$ and $\frac{2}{5}$
8. Find five rational numbers greater than –2
9. Find five rational numbers less than –3

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TO SUM UP

1. Rational numbers are **closed** under **addition, subtraction, multiplication** and **division.**
2. Rational numbers are **commutative** under **addition** and **multiplication.**
3. Rational numbers are **associative** under **addition** and **multiplication.**
4. In rational numbers, there exists an **identity element 'zero'** under addition.
5. In rational numbers, there exists an **identity element 'one'** under multiplication.
6. Given a rational number $\frac{p}{q}$, there exists an additive inverse $-\frac{p}{q}$ such that $\frac{p}{q}+\left(-\frac{p}{q}\right)=0$
7. Given a rational number $\frac{p}{q}$, there exists a multiplicative inverse $\frac{q}{p}$ such that $\frac{p}{q}\times\frac{q}{p}=1$
8. In rational numbers, multiplication, distributes over addition and subtraction.
9. Every rational number can be represented on the number line.
10. On the number line, a rational number on the right is always greater than the number on the left.
11. Between two rational numbers there lie an infinite number of rational numbers.

Sharpen Your Mind

I. Multiple Choice Questions

Choose the correct alternative :

1. The additive inverse of $\frac{-7}{3}$ is ________
   (*a*) $\frac{-3}{7}$ (*b*) $\frac{7}{3}$ (*c*) $\frac{-7}{3}$ (*d*) $\frac{3}{7}$
2. The multiplicative inverse of $\frac{-9}{5}$ is ________
   (*a*) $\frac{-5}{9}$ (*b*) $\frac{-9}{5}$ (*c*) $\frac{9}{5}$ (*d*) $\frac{5}{9}$
3. There are ________ rational numbers between $\frac{-1}{3}$ and $\frac{-1}{4}$
   (*a*) one (*b*) two (*c*) none (*d*) infinite.
4. Rational numbers are commutative under ________
   (*a*) Addition and subtraction only (*b*) Subtraction and multiplication only
   (*c*) Addition and multiplication only (*d*) Subtraction and division only.
5. The product of $\frac{-7}{2}$ and its reciprocal is ________
   (*a*) -1 (*b*) 1 (*c*) $\frac{-49}{4}$ (*d*) $\frac{49}{4}$
6. If *a* and *b* are two rational numbers and $a - b$ is also a rational number, it shows that rational numbers are closed under ________
   (*a*) Addition (*b*) Subtraction (*c*) Multiplication (*d*) Division.
7. Which of the following rational numbers does not lie between -2 and -3?
   (*a*) -2 (*b*) $\frac{-3}{2}$ (*c*) $\frac{-5}{2}$ (*d*) $\frac{-7}{2}$
8. Negative of a negative rational number is ________
   (*a*) Positive (*b*) Negative (*c*) Number itself (*d*) None.
9. Every integer is a ________
   (*a*) Whole number (*b*) Rational number (*c*) Natural number (*d*) None.
10. $\frac{-15}{2} \div \frac{-3}{2} =$ ________
   (*a*) 5 (*b*) -5 (*c*) $\frac{45}{4}$ (*d*) $\frac{-45}{4}$

II. Fill in the Blanks :

1. The numbers ________ and ________ are their own reciprocals.
2. The product of a number and its reciprocal is always ________
3. The sum of a number and its negative is always ________

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4. ________ is the multiplicative identity.

5. ________ is the additive identity.

III. True or False

1. Rational numbers are not closed under multiplication. [ ]
2. Rational numbers are always associative under division. [ ]
3. There are only two rational numbers lying between 1 and 4. [ ]
4. The negative of a positive rational number is positive. [ ]
5. The product of a number and its reciprocal is either 1 or –1. [ ]

IV. Match the Following :

Column A	Column B
(*a*) $(a - b)$ is a rational number	(*i*) Distributive property
(*b*) $a + 0 = 0 + a = a$	(*ii*) Associative property
(*c*) $a \times (b + c) = a \times b + a \times c$	(*iii*) Closure property
(*d*) $a \times b = b \times a$	(*iv*) Commutative property
(*e*) $a + (b + c) = (a + b) + c$	(*v*) Additive identity

Chapter Assessment

M.M. : 30 marks

SECTION A : **Short Answer Questions** (2 marks) **5 × 2 = 10 marks**

1. Write the additive inverse of $\left[\left(\frac{6}{5}+\frac{2}{3}\right)\times\left(\frac{-3}{4}\right)\right]$
2. Divide : $-\frac{5}{4}$ by $\frac{25}{24}$
3. What should be added to $\frac{-9}{7}$ to get $\frac{16}{5}$?
4. Find three rational numbers between $\frac{-3}{7}$ and $\frac{-3}{4}$.
5. Represent $\frac{16}{-5}$ on the number line.

SECTION B : **Short Answer Questions** (3 marks) **4 × 3 = 12 marks**

1. The product of two rational numbers is $\frac{117}{40}$. If one of them is $\left(\frac{-13}{5}\right)$, find the other.
2. Divide the sum of $\frac{7}{8}$ and $\frac{15}{24}$ by their difference.
3. A rope of length $71\frac{1}{2}$ m has been cut into 26 pieces. What is the length of each piece?
4. Simplify : $\left(-\frac{3}{5}\right)\times\left(\frac{-10}{9}\right)\times\left(\frac{21}{-4}\right)\times(-6)$

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2 × 4 = 8 marks

SECTION C : Long Answer Questions (4 marks)

1. Simplify the following using distributive property :

   (a) $\frac{3}{5}\times\left(-\frac{2}{3}\right)+\frac{1}{7}\times\frac{3}{5}$ (b) $-\frac{5}{9}\times\frac{4}{15}-\frac{3}{10}\times\frac{-5}{9}$

2. Express $\left(\frac{1}{2}+\frac{3}{4}\right)\div 2$ as a rational number and show that it lies between $\frac{1}{2}$ and $\frac{3}{4}$.

CHALLENGING QUESTION

A frog is there in the well which is 24 m deep. The frog wants to jump out of the well. Everytime he jumps $\frac{2}{3}$ m and falls back by $\frac{1}{4}$ m. In how many jumps will he come out of the well?

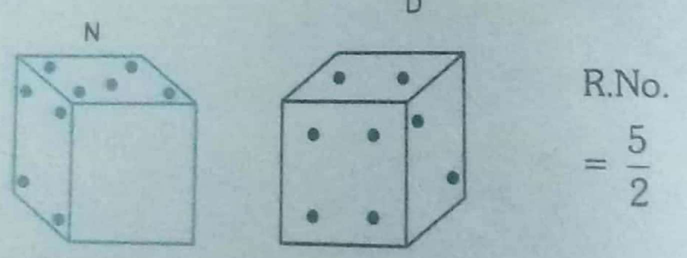
ACTIVITY

Take 2 dice of different colours say red and blue. Throw them together thrice. Consider the number which appears on the red dice, as the numerator and number on the blue dice as denominator. Write these three rational numbers so formed as a, b and c. Are the following properties of rational numbers true?

(a) $a \times (b + c) = a \times b + a \times c$ (b) $a + (b + c) = (a + b) + c$

(c) $a - (b - c) = (a - b) - c$ (d) $a \times (b \times c) = (a \times b) \times c$

(e) $a \div (b \div c) = (a \div b) \div c$



(Red dice) (Blue dice)

Add To Your Knowledge!!

The equals sign (=) was invented in 1557 by a Welsh mathematician named Robert Recorde.